

Online Appendix

Efficient Material Breach of Contract

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1. Baseline Holdup Problem

This section presents the results for the baseline holdup problem with insufficient effort described at the beginning of Section 3. In this scenario, the parties do not enter an ex ante contract, but the seller exerts effort in $t = 2$ and both parties observe the defect δ in $t = 3$ before they negotiate ex post a price p in $t = 4$.

To establish the result of insufficient effort, suppose the buyer makes the ex post contract offer in $t = 4$ with probability 1. This price offer is $\bar{p}_B = 0$. The seller’s expected payoffs are $\pi^S = -c(e)$. The seller’s effort level in this case is $e = 0$. Alternatively, suppose the seller makes the ex post contract offer with probability 1. This price offer is $\bar{p}_S = v - \ell(\delta)$. The seller’s expected payoffs are

$$\begin{aligned}\pi^S &= -c(e) + \int_0^1 [v - \ell(\delta)] f(\delta|e) d\delta \\ &= v - \int_0^1 \ell(\delta) f(\delta|e) d\delta - c(e) = W(e).\end{aligned}\quad (1)$$

The seller’s optimal effort level is $e = e^*$. Finally, suppose the buyer makes the contract offer with probability γ , the seller with $1 - \gamma$. The expected price is $\bar{p} = (1 - \gamma) [v - \ell(\delta)]$. The seller’s expected payoffs are

$$\pi^S = (1 - \gamma) v - (1 - \gamma) \int_0^1 \ell(\delta) f(\delta|e) d\delta - c(e).\quad (2)$$

The seller chooses an effort level $e(\gamma)$ that maximizes π^S . The first-order condition is

$$c'(e) = -(1 - \gamma) \int_0^1 \ell(\delta) f_e(\delta|e) d\delta = (1 - \gamma) \int_0^1 \ell'(\delta) F_e(\delta|e) d\delta.\quad (3)$$

For the expression on the right, I integrate by parts. Because $\ell'(\delta) > 0$ and $F_e(\delta|e) \geq 0$ for all δ , the RHS is strictly positive for $\gamma < 1$. This implies

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that $c'(e) > 0$ and $e(\gamma) > 0$ for $\gamma < 1$. Moreover, for $\gamma > 0$, $e(\gamma) < e^*$ by convexity of $c(e)$.

2. Formal Proofs

This section presents the formal proofs of Lemmata 1 and 2 as well as Propositions 1, 3, and 6.

Proof of Lemma 1. Let $v \geq p$ so that $B_R = v - \beta - p$. First, suppose $\delta \leq \mu$ (trivial breach). The buyer rejects if $B_W > B_A(\delta)$, which is violated for all δ . As consequence, the buyer will not reject (off equilibrium) unless the seller is in material breach. Now, suppose $\delta > \mu$ (material breach). The buyer chooses to reject if $B_R > B_A(\delta)$ or $\alpha(\delta) > \beta$. By Assumption 3, $\bar{\delta}$ denotes the critical value for defect δ such that the buyer rejects if $\delta > \bar{\delta}$ but does not reject otherwise. Combining the two conditions, the buyer rejects (off equilibrium) if and only if $\delta > \max\{\mu, \bar{\delta}\} =: \kappa(\mu)$. Alternatively, let $v < p$ so that $B_R = 0$. If $\delta > \mu$, then the buyer rejects if $B_R > B_A(\delta)$ or $\alpha(\delta) > v - p$. Because $\alpha(\delta) \geq 0$ for all δ and $v - p < 0$, this condition is always true and the buyer always rejects. Because the buyer does not reject when $\delta \leq \mu$, she rejects if and only if $\delta > \mu$ so that $\kappa(\mu) = \mu$. \square

Proof of Lemma 2. Suppose $v \geq p$. Let $\delta \leq \kappa(\mu)$ so that the buyer does not have a credible threat to reject. If the seller accepts the new price, his payoffs are $\bar{p}(\delta)$; if he rejects the price, his payoffs are $p - [\ell(\delta) - \alpha(\delta)]$. The lowest price the buyer can offer in this case of off-equilibrium acceptance (A) is $\bar{p}_A(\delta) = p - [\ell(\delta) - \alpha(\delta)]$. Alternatively, let $\delta > \kappa(\mu)$. If the seller accepts the new price, then his payoffs are $\bar{p}(\delta)$; if he rejects the price, then his payoffs are $-\max\{v - p - \beta, 0\} = p - [v - \beta]$. The lowest price the buyer can offer in this case of off-equilibrium rejection (R) is $\bar{p}_R(\delta) = p - [v - \beta]$. Finally, $\bar{p}_A(\delta) > \bar{p}_R(\delta)$ if $\alpha(\delta) > \beta$ or $\delta > \bar{\delta}$. Because $\bar{\delta} \leq \kappa(\mu)$, if $\delta > \kappa(\mu)$, then $\alpha(\delta) > \beta$.

Instead, let $v < p$. The price for $\delta \leq \kappa(\mu)$ is not affected. Suppose $\delta > \kappa(\mu)$. If the seller accepts the new price, then his payoffs are $\bar{p}(\delta)$; if he rejects the price, then his payoffs are $-\max\{v - p - \beta, 0\} = 0$. The lowest price the buyer can offer in this case of off-equilibrium rejection (R) is $\bar{p}_R(\delta) = 0$. Because $\ell(\delta) - \alpha(\delta) < v$ and $p > v$, $\bar{p}_A(\delta) > 0$ for all δ so that $\bar{p}_A(\delta) > \bar{p}_R(\delta)$ for all δ . \square

Proof of Proposition 1. With $\mu = \kappa(1) = 1$, the seller's payoffs in (14) are

$$p - c(e) - \int_0^1 \ell(\delta) f(\delta|e) d\delta + \int_0^1 \alpha(\delta) f(\delta|e) d\delta. \quad (4)$$

The seller's effort $e = e(\mu)$ satisfies the first-order condition in (16), for $\mu = \kappa(\mu) = 1$:

$$c'(e) = - \int_0^1 [\ell(\delta) - \alpha(\delta)] f_e(\delta|e) d\delta. \quad (5)$$

Because $F_e(\delta|e) \geq 0$, we obtain

$$\int_0^1 \alpha(\delta) f_e(\delta|e) d\delta = - \int_0^1 \alpha'(\delta) F_e(\delta|e) d\delta < 0.$$

By the definition of e^* as solution of the first-order condition in (3), the following holds:

$$c'(e^*) > - \int_0^1 \ell(\delta) f_e(\delta|e^*) d\delta + \int_0^1 \alpha(\delta) f_e(\delta|e^*) d\delta. \quad (6)$$

By convexity of $c(e)$ in e , an effort level $e(1) < e^*$ satisfies the first-order condition in equation (5) when $\mu = 1$. The second-order condition

$$c''(e) = - \int_0^1 [\ell(\delta) - \alpha(\delta)] f_{ee}(\delta|e) d\delta < 0$$

or (through integration by parts)

$$- c''(e) + \int_0^1 [\ell'(\delta) - \alpha'(\delta)] F_{ee}(\delta|e) d\delta - [v - \alpha(1)] F_{ee}(1|e) < 0$$

is satisfied because $\ell'(\delta) - \alpha'(\delta) > 0$ and $F_{ee}(\delta|e) < 0$ for $\delta < 1$ (by Assumption 1(iii)) as well as $F_{ee}(1|e) = 0$ (because $F(1|e) = 1$ for all e). \square

Proof of Proposition 3. For $\mu = \mu^*$ we must have $\kappa(\mu^*) = \mu^*$. For a low price such that $v \geq p$, the condition for μ^* to implement efficient effort is $\sigma(\mu^*|e^*) = 0$, or

$$H := \tilde{\alpha} \int_0^{\mu^*} \ell(\delta) f_e(\delta|e^*) d\delta - \int_{\mu^*}^1 [v - \beta - \ell(\delta)] f_e(\delta|e^*) d\delta = 0 \quad (7)$$

with $\alpha(\delta) = \tilde{\alpha}\ell(\delta)$.

Effect of β on μ^ :* Implicit differentiation yields

$$\frac{d\mu^*}{d\beta} = - \frac{H_\beta}{H_{\mu^*}} < 0$$

where (by integration by parts)

$$H_\beta = \int_{\mu^*}^1 f_e(\delta|e^*) d\delta = F_e(\delta|e^*) \Big|_{\mu^*}^1 = -F_e(\mu^*|e^*) < 0 \quad (8)$$

and

$$H_{\mu^*} = \int_{\mu^*}^1 f_e(\delta|e^*) d\delta + [v - \beta - [\ell(\mu^*) - \alpha(\mu^*)]] f_e(\mu^*|e^*) < 0. \quad (9)$$

For the negative sign of H_{μ^*} , observe that

$$v - \beta - [\ell(\mu^*) - \alpha(\mu^*)] > 0 \iff v - \ell(\mu^*) > 0 > \beta - \alpha(\mu^*).$$

First, $v > \ell(\mu^*)$ for $\mu^* < 1$ and $\alpha(\mu^*) > \beta$ because $\mu^* > \bar{\delta}$ and $\alpha(\delta) > \beta$ when $\delta > \bar{\delta}$. Hence, $v - \beta - [\ell(\mu^*) - \alpha(\mu^*)] > 0$. Moreover, because $f_e(\mu^*|e^*) < 0$ for $\mu^* > \delta^o(e)$, the expression is negative. As a consequence,

$d\mu^*/d\beta < 0$, the threshold of efficient material breach decreases in β . For high values of p such that $v < p$, $H_\beta = 0$.

Effect of $\tilde{\alpha}$ on μ^ :* Suppose low p is such that $v \geq p$. Implicit differentiation yields

$$\frac{d\mu^*}{d\tilde{\alpha}} = -\frac{H_{\tilde{\alpha}}}{H_{\mu^*}}$$

with

$$\begin{aligned} H_{\tilde{\alpha}} &= \int_0^{\mu^*} \ell(\delta) f_e(\delta|e^*) d\delta \\ &= \frac{\int_{\mu^*}^1 [v - \beta - \ell(\delta)] f_e(\delta|e^*) d\delta}{\tilde{\alpha}} \end{aligned} \quad (10)$$

$$= \frac{-[v - \beta - \ell(\mu^*)] F_e(\mu^*|e^*) + \int_{\mu^*}^1 \ell'(\delta) F_e(\delta|e^*) d\delta}{\tilde{\alpha}}. \quad (11)$$

The equality of (10) is by definition of $\sigma(\mu^*|e^*)$, and the expression for (11) is by integration by parts with $F_e(1|e^*) = 0$. Note that for $\beta = 0$, the expression in (10) is negative because $v - \ell(\delta) \geq 0$ for all δ and $f_e(\delta|e^*) < 0$ for all $\delta > \mu^* > \delta^o(e)$. For $\beta = v - \ell(\mu^*)$, the expression in (11) is positive because $\ell'(\delta) \geq 0$ and $F_e(\delta|e^*) \geq 0$ for all δ . By continuity of (11) in β , there is some $\tilde{\beta} \in (0, v - \ell(\mu^*))$ such that $H_{\tilde{\alpha}} = 0$ for $\beta = \tilde{\beta}$, $H_{\tilde{\alpha}} < 0$ for $\beta < \tilde{\beta}$ and $H_{\tilde{\alpha}} > 0$ for $\beta > \tilde{\beta}$. As a consequence, $d\mu^*/d\tilde{\alpha} < 0$ for low β and $d\mu^*/d\tilde{\alpha} > 0$ for high β .

Suppose high p such that $v < p$. Implicit differentiation of

$$\begin{aligned} H &:= \frac{\partial \sigma(\mu, p|e^*)}{\partial e} \\ &= \tilde{\alpha} \int_0^{\mu^*} \ell(\delta) f_e(\delta|e^*) d\delta - \int_{\mu^*}^1 [p - \ell(\delta)] f_e(\delta|e^*) d\delta = 0 \end{aligned} \quad (12)$$

yields

$$\frac{d\mu^*}{d\tilde{\alpha}} = -\frac{H_{\tilde{\alpha}}}{H_{\mu^*}}$$

with H_{μ^*} as in (9) (for $p = v - \beta$) and

$$H_{\tilde{\alpha}} = \frac{\int_{\mu^*}^1 [p - \ell(\delta)] f_e(\delta|e^*) d\delta}{\tilde{\alpha}} > 0 \quad (13)$$

because $p > \ell(\delta)$ for all δ and $f_e(\delta|e^*) < 0$ for all $\delta > \mu^* > \delta^o(e)$.

Effect of p on μ^ :* For $v \geq p$, the condition for μ^* in (7) does not depend on p so that μ^* does not change with p . For $v < p$, the condition for μ^* is

$$\begin{aligned} H &:= -c'(e^*) - \int_0^1 [\ell(\delta) - \alpha(\delta)] f_e(\delta|e^*) d\delta \\ &\quad - \int_{\mu^*}^1 [p - [\ell(\delta) - \alpha(\delta)]] f_e(\delta|e^*) d\delta = 0. \end{aligned}$$

Implicit differentiation yields

$$\frac{d\mu^*}{dp} = -\frac{H_p}{H_{\mu^*}} = -\frac{-\int_{\mu^*}^1 f_e(\delta|e^*)d\delta}{[p - [\ell(\mu^*) - \alpha(\mu^*)]] f_e(\mu^*|e^*)} = -\frac{(+)}{(-)} > 0. \quad (14)$$

The denominator is negative because

$$p - [\ell(\mu^*) - \alpha(\mu^*)] > v - [\ell(\mu^*) - \alpha(\mu^*)] > 0.$$

The numerator is positive because $f_e(\delta|e^* < 0)$ for all $\delta \geq \mu^* > \delta^o(e)$. \square

Proof of Proposition 6. See Figure 3. For all $p < \bar{p}$, the efficient outcome cannot be implemented in a regime of rejection without damages. In a regime of rejection with damages, the efficient outcome can be implemented for sufficiently low β . Because $\beta \leq \max\{v - p, 0\}$, under-compensation in this latter regime is not more stringent than in the former. \square

3. Efficient Material Breach Without Renegotiation

In this section, I demonstrate the effect of a satisfaction clause when the parties cannot renegotiate the ex ante contract. For this treatment, I assume that the price is relatively low (i.e., $p \leq v$) and that the baseline probability of no defect is zero (i.e., $h(e) = F(0|e) = 0$ and $h_e(e) = F_e(0|e) = 0$).

First, by Lemma 1, the buyer rejects delivery when it is profitable to do so, that means, for sufficiently high defects $\delta > \kappa(\mu)$. Unlike in the renegotiation scenario where the buyer's rejection is only a threat and the parties always agree on a new contract price ex post so that trade always occurs, now the buyer rejects and returns the seller's delivery in equilibrium if $\mu < 1$.

To construct the seller's expected payoffs under a simple contract $\langle p, \mu \rangle$, we first need to establish the seller's payoffs under rejection and no rejection. For large enough defects so that the buyer rejects, the seller's payoffs are as in equation (8): $S_R(e) = -\max\{v - \beta(\delta) - p, 0\} - c(e)$. If the realized defect is low so that the buyer accepts delivery, then the seller's payoffs are as in equation (6): $S_A(e, \delta) = p - c(e) - [\ell(\delta) - \alpha(\delta)]$. The seller's expected payoffs from an ex ante contract are then

$$\pi_{NR}^S = \mathbb{E}_\delta [S_A(e, \delta) | \delta \leq \kappa(\mu)] + \mathbb{E}_\delta [S_R(e, \delta) | \delta > \kappa(\mu)].$$

After some rearranging, this expression can be rewritten

$$\pi_{NR}^S = p - \int_0^1 \ell(\delta) f(\delta|e) d\delta - \Phi_{NR}(e, \mu)$$

where

$$\Phi_{NR}(e, \mu) = c(e) - \int_0^{\kappa(\mu)} \alpha(\delta) f(\delta|e) d\delta + \int_{\kappa(\mu)}^1 [v - \beta - \ell(\delta)] f(\delta|e) d\delta.$$

Observe that these expressions are the same as the ones for the seller's expected payoffs in the scenario with renegotiation (equations (14) and (15)),

$\pi^S = \pi_{NR}^S$. As a consequence, the seller's effort in the scenario without renegotiation is the same as the in the scenario with renegotiation. Note that this is an artifact of the assumption that the buyer makes the ex post renegotiation offer. If, instead, the seller were to make an offer with strictly positive probability, then this equivalence ceases to hold. The qualitative results, however, are unaffected by the details of the ex post bargaining protocol in the scenario with renegotiation.

The buyer's payoffs when he rejects (for high enough δ) are as in equation (7): $B_R = \max\{v - \beta(\delta) - p, 0\}$. If the realized defect is low so that the buyer accepts delivery, then his payoffs are as in equation (5): $B_A(\delta) = v - \alpha(\delta) - p$. His expected payoffs are then

$$\pi_{NR}^B = \mathbb{E}_\delta [B_A(\delta) | \delta \leq \kappa(\mu)] + \mathbb{E}_\delta [B_R | \delta > \kappa(\mu)]. \quad (15)$$

After some rearranging, this expression can be rewritten as

$$\pi_{NR}^B = \Psi_{NR}(e, \mu) - p \quad (16)$$

with

$$\Psi_{NR}(e, \mu) = v - \int_0^{\kappa(\mu)} \alpha(\delta) f(\delta|e) d\delta - \int_{\kappa(\mu)}^1 \beta f(\delta|e) d\delta \quad (17)$$

The parties' joint expected surplus is equal to the sum of their individual expected payoffs: $W_{NR}(e, \mu) = \pi_{NR}^S = \pi_{NR}^B$ or

$$\begin{aligned} W_{NR}(e, \mu) &= \Psi_{NR}(e, \mu) - \Phi_{NR}(e, \mu) - \int_0^1 \ell(\delta) f(\delta|e) d\delta \\ &= W(e) - \int_{\kappa(\mu)}^1 [v - \ell(\delta)] f(\delta|e) d\delta. \end{aligned} \quad (18)$$

For a given effort level e , these joint payoffs are equal to the gains from trade, $W(e)$, minus the social loss from returning the good (i.e., the buyer's valuation of the seller's delivery). As a consequence, whenever the buyer is granted the right to reject (and chooses to exercise her option with strictly positive probability when $\kappa(\mu) < 1$), we have

$$W_{NR}(e, \mu) < W(e). \quad (19)$$

As I establish in Proposition 7 below, the optimal satisfaction clause allows the buyer to reject and return the delivery with strictly positive probability. This implies that even when rejection comes at a social cost of no trade, the parties are willing to incur these costs in exchange for higher effort that is induced by $\mu < 1$.

Proposition 7. For β sufficiently low, when renegotiation of the contract is not possible, the ex ante contract specifies a unique satisfaction clause $\mu_{NR} < 1$.

Proof. Suppose that β is small so that both $\kappa(\mu) = \mu$. For higher β (so that $\bar{\delta} > \mu$, the satisfaction clause is ineffective). The first-order condition for μ_{NR} to maximize the joint payoffs $W_{NR}(e, \mu)$ is

$$\frac{dW(e)}{de} \frac{\partial e(\mu)}{\partial \mu} - \left[\int_{\mu}^1 [v - \ell(\delta)] f_e(\delta|e) d\delta \cdot \frac{\partial e(\mu)}{\partial \mu} - [v - \ell(\mu)] f(\mu|e) \right] = 0. \quad (20)$$

Because the seller's problem in this scenario without renegotiation is the same as in the scenario with renegotiation, we can use the results from the latter. Recall the first-order condition for seller's effort in (16):

$$H := -c'(e) - \int_0^1 \ell(\delta) f_e(\delta|e) d\delta + \left[\int_0^{\kappa(\mu)} \alpha(\delta) f_e(\delta|e) d\delta - \int_{\kappa(\mu)}^1 [v - \beta - \ell(\delta)] f_e(\delta|e) d\delta \right].$$

Implicit differentiation yields

$$\frac{\partial e(\mu)}{\partial \mu} = -\frac{H_{\mu}}{H_e} = -\frac{[v - \beta - [\ell(\mu) - \alpha(\mu)]] f_e(\mu|e)}{\text{(second-order condition)}} = -\frac{(-)}{(-)} < 0. \quad (21)$$

The numerator is negative for all μ as shown in the proof of Proposition 3; the denominator is negative by assumption of the second-order condition satisfied. Evaluating condition (20) at $\mu = 1$ yields

$$\left. \frac{dW(e)}{de} \frac{\partial e(\mu)}{\partial \mu} \right|_{\mu=1} < 0 \quad (22)$$

because: (i) $\frac{\partial e(\mu)}{\partial \mu} < 0$ when evaluated at $\mu = 1$; (ii) $e(1) < e^*$ so that $\frac{dW(e)}{de} > 0$ when evaluated at $e(1)$; and (iii) $v - \ell(1) = 0$. As a result, the optimal threshold of material breach is strictly less than unity and therefore allows for *some* rejection. Unlike in the case with renegotiation, the optimal threshold of material breach, $\mu_{NR} < 1$, is unique. If there are two values that induce the same level of effort, then the higher value results in a smaller loss from gains from trade and dominates the lower value. \square

At the contracting stage $t = 2$, the parties now face the following tradeoff. To minimize the joint losses from returned goods, they are inclined to agree on a satisfaction clause that never allows the buyer to reject and return (i.e., $\mu = 1$). Such a satisfaction clause, however, induces the seller to exert inefficient effort (because of imperfect compensation). This low level of effort increases the expected losses from defective delivery when the buyer accepts. As consequence, to induce the seller to exert more effort and reduce the expected defect, the optimal contract incorporates a satisfaction clause that allows the buyer to reject for some defects—at the cost of returned goods.

4. Under-Compensation with Limited Verifiability of Defects

In the main text of the paper (Section 3), I motivate under-compensation as follows. Enforcement imperfections of the default breach remedies (i.e., under-compensation) arise because the true losses $\ell(\delta)$ cannot be verified in court, whereas the defect δ is verifiable (and thus contractible). For the analysis of liquidated damages (Section 4 and the next section in the Online Appendix), I utilize a different framework to motivate under-compensation. I assume that enforcement imperfections arise because the defect δ cannot be verified in court with certainty.

Suppose the parties expect the buyer (upon acceptance, A) to be unable to show a defect δ with reasonable certainty with probability $\bar{\alpha}$. As a result, her *expected* expectation damages are $(1 - \bar{\alpha})\ell(\delta)$ and the expected under-compensation is $\alpha(\delta) = \bar{\alpha}\ell(\delta)$. A stricter standard of proof (i.e., a lower probability with which the buyer can show the defect) implies a higher value of $\bar{\alpha}$.

If the buyer chooses to reject the seller's delivery (R), she must first show (with reasonable certainty) that the seller is indeed in material breach. Suppose $\bar{\beta}$ is the probability that the buyer is unable to convince the court that $\delta > \mu$. In other words, rejection is wrongful with probability $\bar{\beta}$. Suppose this probability is high for small defects δ and low with larger defects, implying that $\bar{\beta}(\delta)$ decreases in the value of δ . For instance, let $\bar{\beta}(\delta) = 1$ if $\delta \leq \mu$ and $\bar{\beta}(\delta) < 1$ and decreasing for all $\delta > \mu$.

The buyer's payoffs upon rightful rejection are equal to $B_R = \max\{v - p, 0\}$, and equal to $B_W = \omega[v - \ell(\delta)] - p$ as in equation (9) for wrongful rejection. Her expected payoffs from rejection are then equal to

$$B_{RW} = \begin{cases} v - p - \bar{\beta}(\delta)[v - \omega(v - \ell(\delta))] & \text{if } v \geq p \\ -\bar{\beta}(\delta)[p - \omega(v - \ell(\delta))] & \text{if otherwise.} \end{cases}$$

This yields under-compensation (in a regime of rejection with damages), $v - p - B_{RW}$, of

$$\beta(\delta) = \begin{cases} \bar{\beta}(\delta)[v - \omega(v - \ell(\delta))] & \text{if } v \geq p \\ v - p + \bar{\beta}(\delta)[p - \omega(v - \ell(\delta))] & \text{if otherwise.} \end{cases} \quad (23)$$

In a regime of rejection without damages, the buyer's effective under-compensation from rejection is

$$\beta(\delta) = v - p + \bar{\beta}(\delta)[p - \omega(v - \ell(\delta))]. \quad (24)$$

Because by Assumption 2, $\alpha(\delta) = \bar{\alpha}\ell(\delta)$ increases in δ whereas $\bar{\beta}(\delta)$ decreases in δ , Assumption 3 holds (and a critical value $\bar{\delta} < 1$ such that $\alpha(\delta) \geq \beta$ for all $\delta \geq \bar{\delta}$ exists) for sufficiently high values of ω or sufficiently low values of $\bar{\beta}$. As a consequence, the results from Section 3 in the main text carry over to this model of under-compensation.